International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 08, August 2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

FRACTIONAL DERIVATIVES OF MULTIVARIABLE H-FUNCTION AND THEIR APPLICATIONS

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Abstract

In this paper we use fractional differential operators $D^{n}_{k,\alpha,x}$ and ${}_{\alpha}D^{\mu}_{x}$ to derive a number of key formulas of multivariable H-function. We use the generalized Leibnitz's rule for fractional derivatives in order to obtain one of the aforementioned formulas, whichinvolve a product of two multivariables H-function. It is further shown that ,each of these formulas yield interesting new formulas for certain multivariable hypergeometric function such as generalized Lauricella function (Srivastava-Dauost)and Lauriella hypergeometric function some of these application of the key formulas provide potentially useful generalization of known result in the theory of fractional calculus.

Key words:-Fractional differential operator, multivariable H-function.
AMS subjectclassification 2010 MSC: 26A33 ,33C20

INTRODUCTION AND DEFINITIONS

The fractional derivative of special function of one and more variables is important such as in the evaluation of series,[10,15] the derivation of generating function [12,chap.5] and the solution of differential equations [4,14;chap-3] motivated by these and many other avenues of applications, the fractional differential operators $D^{n}_{k,\alpha,x}$ and ${}_{\alpha}D^{\mu}_{x}$ are much used in the theory of special function of one and more variables .

We use the fractional derivative operator defined in the following manner [16]

$$D^{n}_{k,\alpha,x}(x^{\mu}) = \prod_{r=0}^{n-1}\left[\frac{\sqrt{\mu+rk+1}}{\sqrt{\mu+rk-\alpha+1}}\right] x^{\mu+nk} \quad \ldots (1.1)$$

Whereα≠μ+1 andα and k are not necessarily integers

We use the binomial expansion in the following manner

$(ax^{\mu}+b)^{\lambda}=b^{\lambda}\sum_{l=0}^{\infty}\binom{\lambda}{l}\left(\frac{ax^{\mu}}{b}\right)^{l}$ where $[\frac{ax^{\mu}}{b}]<1$... (1.2) the familiar differential operator ${}_{\alpha}D_{x}^{\mu}$ is defined by [5, p.49; 3; 9; 17, P-356]

$$
{}_{\alpha}D_{x}^{\mu}f(\mathrm{x})=\begin{cases}\dfrac{1}{\sqrt{-\mu}}\displaystyle\int_{\alpha}^{x}(x-t)^{-\mu-1}f(t)dt\ , & [\mathrm{Re}(\mu)<0]\\ \dfrac{d^{m}}{dx^{m}}\,{}_{\alpha}D_{x}^{\mu-m}f(x), & [0\le\mathrm{Re}(\mu)<\mathrm{m}]\end{cases}
$$

...(1.3)

Where m is a positive integer

For $\alpha=0$,(1.3) Defines the classical Riemann-Liouville fractional derivative of order μ (or-μ) when α→∞ (1.3) may be identified with the definition of the well known Weyl fraction derivative of order μ (or-μ) [1,chap.13);3] the special case of fractional calculus operator ${}_{\alpha}D_{x}^{\mu}$ when α=0 is written as D_{x}^{μ} thus we have

$$
D_{x}^{\mu}={}_{0}D_{x}^{\mu} \qquad \text{... (1.4)}
$$

In this paper we drive several fractional derivative formulas involving multivariable H-function which as defined by srivastav and panda [8, p.271 (4.1) et. Seq.] and studied systematically by then [6,7,8 also 11] for this multivariable H-function we adopt the contracted notations (due essentially to Srivastava and panda [13] thus following the various conventions and notations explained fairly and fully in their earlier works [6,7,8see also 11,13]

$$
H[z_1,\ldots,z_r]=H_{p,q:\,p_1,\,q_1,\ldots,p_r,\,q_r}^{0,n:\,m_1,\,n_1,\ldots,m_r,\,n_r}\left[\begin{matrix}z_1\\ \cdot\\ \vdots\\ z_r\end{matrix}\left|\begin{matrix}\left(a_j,\alpha_j^{(1)}\ldots\alpha_j^{(r)}\right)_{1,p}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1},\ldots,\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\\ \\ \left(b_j,\beta_j^{(1)}\ldots\beta_j^{(r)}\right)_{1,q}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1},\ldots,\left(d_j^{r},\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right.\right] \qquad \text{...(1.5)}
$$

Denote the H-function of r-variables $z_1,z_2,\ldots,z_r$ here for convenience

$\left(a_{j,}\alpha_j^1,...\alpha_j^{(r)}\right)_{1,p}$ Abbreviates the p- member array

$\left(a_{1,}\alpha_1^1,...\alpha_1^{(r)}\right),...,\left(a_{p,}\alpha_p^1,...\alpha_p^{(r)}\right)$

While $\left(c_j^{(i)},\gamma_j^{(i)}\right)_{1,p_2}$ Abbreviates the array of p_i pairs of parameters

$$\left(\alpha_j^{(i)},\gamma_j^{(i)}\right),.............,\left(\alpha_{p_i}^{(i)},\gamma_{p_i}^{(i)}\right) \quad ; \quad (i=1,...,r) \qquad ... (1.7)$$

and so on ,suppose , asusual that the parameters

$$a_j\,,\quad j=1,...,p;\qquad c_j^{(i)},\quad j=1,...,p_i;$$

$$b_j\,,\quad j=1,...,\mathrm{q};\qquad d_j^{(i)},\ j=1,...,q_i\,;\qquad \forall i\in(i=1,...,r) \qquad ...(1.8)$$

Are complex number and the associated coefficients

$$\alpha_{j,}\quad j=1,...,p;\ \ \gamma^{(i)}{}_j\ ,\ j=1...p_i\,;$$

$$\beta_j,\ j=1,...,q;\ \delta^{(i)}{}_j\,,\ \ j=1...\mathrm{q}_i\ ;\qquad \forall i\in(1,...,r)$$

... (1.9)

Arepositive real numbers such that

$$\Lambda_i=\sum_{j=1}^{r}\alpha_j^{(i)}-\sum_{j=1}^{q}\beta_j^{(i)}+\sum_{j=1}^{p_i}\gamma_j^{(i)}-\sum_{j=1}^{q_i}\delta_j^{(i)}\le 0 \qquad ... (1.10)$$

and

$$\Omega_i=-\sum_{j=n+1}^{p}\alpha_j^{(i)}-\sum_{j=1}^{q}\beta_j^{(i)}+\sum_{j=1}^{n_i}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=m_2+1}^{q_i}\delta_j^{(i)}>0\ ;\ \ \forall i\in[1,...,r] \qquad ... (1.11)$$

Where the integers n, p, q, m_i, n_i, p_i, q_i are constrained by the inequalities $0\le n\le p,\ q\ge 0$, $1\le m_i\le q_i, 0\le n_i\le p_i$ $[i=1,...,r]$ and the equality (1.10) holds true for suitably restricted values of the complex variables $z_1,.....,z_r$

Then it is known that the multiple Mellin-Barnes counter integral [11, p.251 (c.1)] representing the multivariable H-function (1.5) converges absolutely under the condition (1.11) when

$$\left|\arg\left(z_i\right)\right| < \frac{1}{2}\Lambda\Omega_i \quad , \ \forall i \in \left[1,\ldots,r\right] \ldots (1.12)$$

$$H\left[z_1,\ldots z_r\right] = \left| \begin{matrix} 0\left(\left|z_1\right|^{\xi_1} \ldots, \left|z_r\right|^{\xi_r}\right), & \left(\max\left|z_1\right|,\ldots,\left|z_r\right| \to 0\right) \\ 0\left(\left|z_1\right|^{\eta_1} \ldots, \left|z_r\right|^{\eta_r}\right), & \left(\eta = 0; \min\left|z_1\right|,\ldots,\left|z_r\right| \to \infty\right) \end{matrix} \right|$$

... (1.13)

Where with i=1,…,r

$$\xi_i = \min\left\{\operatorname{Re}\left(d_j^{(i)} / \delta_j^{(i)}\right)\right\}, \qquad \left(j = 1,\ldots,m_i\right)$$

$$\eta_i = \max\left\{\operatorname{Re}\left(c_j^{(i)} - 1 / \gamma_j^{(i)}\right)\right\}, \qquad \left(j = 1,\ldots,n_i\right) \qquad \ldots (1.14)$$

Provided that each of the inequalities (1.10) (1.11) and (1.12) holds true.

Throughout the present paper .we assume that the convergence and existence condition corresponding appropriately to the ones detained above are satisfied by each of the various H-function involved in our results which are presented in the following sections

2. MAIN RESULT

In this section we shall prove our main formulas on fractional differential operator involving multivariable H-function

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1. RESULT

$$D_{k,\alpha,x}^{n}\{x^{t}\left(x^{\upsilon_1}+a\right)^{\lambda}\left(b-x^{\upsilon_2}\right)^{-\delta}$$

$$H[z_1 x^{\rho_1}\left(x^{\upsilon_1}+a\right)^{\sigma_1}\left(b-x^{\upsilon_2}\right)^{-\delta},\ldots,z_r\, x^{\rho_r}\left(x^{\upsilon_1}+a\right)^{\sigma_r}\left(b-x^{\upsilon_2}\right)^{-\delta_r}]\}$$

$$=a^{\lambda}b^{-\delta}x^{t+nk}\sum_{l,m=0}^{\infty}\frac{\left(x^{\upsilon_1}/a\right)^{l}\left(x^{\upsilon_2}/b\right)^{m}}{l!m!}H_{p'+n+2,q'+n+2:p'_1,q'_1;\ldots;p'_r,q'_r}^{0,n'+n+2:m'_{1,}n'_1;\ldots;m'_r,n'_r}$$

$$\left[\begin{matrix} z_1 x^{\rho_1}a^{\sigma_1}b^{-\delta_1} \\ \vdots \\ z_r x^{\rho_r}a^{\sigma_r}b^{-\delta_r}\end{matrix}\left|\begin{matrix}\left(-\lambda,\sigma_{1.},\ldots,\sigma_r\right),\left(1-\delta-m,\delta_1,\ldots,\delta_r\right),\left(-t-gk-\upsilon_1 l-\upsilon_2 m;\rho_1,\ldots,\rho_r\right)_{g=0,n-1},\left(a_j,\alpha_j^{(1)}\ldots\alpha_j^{(r)}\right)_{1,p'}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p'_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p'_r} \\ \\ \left(-\lambda+l,\sigma_1,\ldots\sigma_r\right),\left(1-\delta,\delta_1,\ldots\delta_r\right),\left(\alpha-t-gk-\upsilon_1 l-\upsilon_2 m;\rho_1,\ldots\rho_r\right)_{g=0,n-1},\left(b_j,\beta_j^{(1)}\ldots\beta_j^{(r)}\right)_{1,q'}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q'_1};\ldots;\left(d_j^{r},\delta_j^{(r)}\right)_{1,q'_r}\end{matrix}\right.\right]$$

… (2.1)

Provided (in addition to the appropriate convergence and existence condition) that

$$\min\left\{\upsilon_{1,}\upsilon_2,\rho_{i,}\sigma_i,\delta_i\right\}>0\ \left(i=1,\ldots,r\right);$$

$$\max\left\{\left|\arg\left(x^{\upsilon_1}/a\right)\right|,\left|\arg\left(x^{\upsilon_2}/b\right)\right|\right\}<\pi$$

$$\operatorname{Re}\left(k\right)+\sum_{i=1}^{r}\rho_i\xi_i>-1$$

Where $\xi_i=\left(i=1,\ldots,r\right)$ are given in (1.14)

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2.RESULT

$$D_{k,\alpha,x}^{n}\, D_{y}^{\mu}\, \{ x^{t} y^{\lambda} \left(x^{\upsilon_1}+a\right)^{\lambda} \left(b-x^{\upsilon_2}\right)^{-\delta} \left(y^{\upsilon_3}+c\right)^{h} \left(d-y^{\upsilon_4}\right)^{-g}$$

$$H\,[z_1\, x^{\rho_1}\, y^{\lambda_1} \left(x^{\upsilon_1}+a\right)^{\sigma_1} \left(b-x^{\upsilon_2}\right)^{-\delta_1} \left(y^{\upsilon_3}+c\right)^{h_1} \left(d-y^{\upsilon_4}\right)^{-g_1} \ldots\ldots\ldots$$

$$z_r\, x^{\rho_r}\, y^{\lambda_r} \left(x^{\upsilon_1}+a\right)^{\sigma_r} \left(b-x^{\upsilon_2}\right)^{-\delta_r} \left(y^{\upsilon_3}+c\right)^{h_r} \left(d-y^{\upsilon_4}\right)^{-g_r}\,]\}$$

$$= a^{\lambda} b^{-\delta} c^{h} d^{-g} x^{t+nk} y^{\lambda-\mu} \sum_{l,m,r,s=0}^{\infty} \frac{\left(x^{\upsilon_1}/a\right)^{l} \left(x^{\upsilon_2}/b\right)^{m} \left(y^{\upsilon_3}/c\right)^{r} \left(y^{\upsilon_4}/d\right)^{s}}{l!\,m!\,\mathrm{r}!\,\mathrm{s}!}\; H^{\,0,\,n'+n+5:\, m'_1, n'_1, \ldots, m'_r, n'_r}_{\,p'+n+5,\, q'+n+5:\, p'_1, q'_1, \ldots,\, p'_r, q'_r}$$

$$\left[\begin{matrix} z_1 x^{\rho_1} y^{\lambda_1} a^{\sigma_1} b^{-\delta_1} c^{h_1} d^{-g_1} \\ \vdots \\ z_r x^{\rho_r} y^{\lambda_r} a^{\sigma_r} b^{-\delta_r} c^{h_r} d^{-g_r} \end{matrix}\right| \begin{matrix} \left(-\sigma,\sigma_{1.},\ldots,\sigma_r\right),\left(1-\delta-m,\delta_1,\ldots,\delta_r\right),\left(-h,h_1,\ldots.h_r\right),\left(1-g-s,g_1,\ldots g_r\right), \\ \\ \left(-\sigma+l,\sigma_1,\ldots\sigma_r\right),\left(1-\delta,\delta_1,\ldots\delta_r\right),\left(-h+r,h_{1,}\ldots.h_r\right),\left(1-g,g_1,\ldots g_r\right), \end{matrix}$$

$$\left.\begin{matrix} \left(-\lambda-r\upsilon_3-s\upsilon_4,\lambda_1,\ldots.\lambda_r\right),\left(-t-gk-\upsilon_1 l-\upsilon_2 m;\mathrm{k}_1,\ldots k_r\right)_{g=0,\mathrm{n}-1},\left(a_j,\alpha_j^{(1)}\ldots\alpha_j^{(r)}\right)_{1,p'}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p'_1}\cdots\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p'_r} \\ \\ \left(-\lambda+\mu-r\upsilon_3-s\upsilon_4,\lambda_1,\ldots.\lambda_r\right),\left(\alpha-t-gk-\upsilon_1 l-\upsilon_2 m;\mathrm{k}_1\ldots\mathrm{k}_r\right)_{g=0,n-1},\left(b_j,\beta_j^{(1)}\ldots\beta_j^{(r)}\right)_{1,q'}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q'_1}\cdots\left(d_j^{r},\delta_j^{(r)}\right)_{1,q'_r} \end{matrix}\right]$$

.... (2.2)

Provided (in addition to the appropriate convergence and existenceconditions that

$$\min\left(\upsilon_1,\upsilon_2,\upsilon_3,\upsilon_4,\sigma_i,\delta_i,h_i,g_i\right)>0\ \left(i=1,\ldots,\mathrm{r}\right)$$

$$\max\left\{\left|\arg\left(x^{\upsilon_1}/a\right)\right|,\left|\arg\left(x^{\upsilon_2}/b\right)\right|,\arg\left|\left(y^{\upsilon_3}/c\right)\right|,\arg\left|\left(y^{\upsilon_4}/d\right)\right|\right\}<\pi$$

$\mathrm{Re}(k)+\sum_{i=1}^{r}k_i\xi_i > -1$ $\mathrm{Re}(\lambda)+\sum_{i=1}^{r}\lambda_i\xi_i > -1$ And where $\xi_1,\ldots\xi_r$ are given in (1.14)

Proof of (2.1) :-

Wefirst replace the multivariable H-function occurring on the LHS by its Mellin Barnes contour integrals collected the powers of x, $\left(x^{\upsilon_1}+a\right)$ and $\left(b-x^{\upsilon_2}\right)$ and apply binomial expansion

$$\left(x+\xi\right)^{\lambda}=\xi^{\lambda}\left(1+\frac{x}{\xi}\right)^{\lambda}=\xi^{\lambda}\sum_{l=0}^{\infty}\binom{\lambda}{l}\left(\frac{x}{\xi}\right)^{l}; \qquad \left|\frac{x}{\xi}\right|<1 \ldots (2.3)$$

We then apply the fractional derivative operator in the following manner [2]

$$D_{k,\alpha,x}^{n}\left(x^{\mu}\right)=\prod_{r=0}^{n-1}\left[\frac{\Gamma\mu+rk+1}{\Gamma\mu+rk-\alpha+1}\right]x^{\mu+nk} \ldots (2.4)$$

Where $\alpha\neq\mu+1$ and α and k are not necessarily integers and interpret the resulting MillenBarnes contour integrals as a H-function of r-variables we shall arrive at(2.1)

Proof of (2.2):-

Wefirst replace the multivariable H-function occurring on the LHS by its Mellin-Barnesintegrals Collected the powers of x, y, $\left(x^{\upsilon_1}+a\right),\left(b-x^{\upsilon_2}\right),\left(y^{\upsilon_3}+c\right),\left(d-y^{\upsilon_4}\right)$ and apply the binomial

$$\left(x+\xi\right)^{\lambda}==\xi^{\lambda}\sum_{l=0}^{\infty}\binom{\lambda}{l}\left(\frac{x}{\xi}\right); \qquad \left|\frac{x}{\xi}\right|<1$$

We then apply the formula [7, p.67 eq.4.4.4]

$$D_x^{\mu}\left(x^{\lambda}\right)=\frac{\Gamma 1+\lambda}{\Gamma 1+\lambda-\mu}x^{\lambda-\mu}, \quad \left[\mathrm{Re}(\lambda)>-1\right]$$

and

$$D_{k,\alpha,x}^{n}\left(x^{\mu}\right)=\prod_{r=0}^{n-1}\left[\frac{\Gamma\mu+rk+1}{\Gamma\mu+rk-\alpha+1}\right]x^{\mu+nk}$$

Where $\alpha\neq\mu+1$ and α and k are not necessarily integers and interpret the resulting Mellin-Barnes contour integrals as a H-function of r-variables we shall arrive at(2.2)

3. Applications

Each fractional derivative formula (2.1) and (2.2) has manifold generality. By specializing the various parameters and variables involved , these formulas (and indeed their several variations obtained by letting any desired number of the exponents: $\rho_1,...,\rho_r;\sigma_1,...,\sigma_r;\delta_1,...,\delta_r;\mathrm{h}_1,...,h_r;g_1,...,g_r;\lambda_1,...,\lambda_r$ decrease to zero in such a manner that both the sides of the resulting equation exit) can suitably be applied to derive the corresponding results involving a wide variety of useful functions (or product of several such functions) which are expressible in terms of the E,F,G and H-functions of one ,two and more variables. Say, if we put n = p = q = 0 ,the multivariable H-functions occurring on the LHS of (2.1) and (2.2) would immediately reduce to the product r(or s) different Fox's H- functions. Various special cases of Fox's H-function can be seen in a monograph of Mathai and Saxena [18, p.145-159].Thus it can also easily be derived fractional derivative formulas involving any of these simpler special functions desired.

(1) Replacing δ by $(-\delta)$ in (2.1) and (2.2) ; setting $\sigma_i,\delta_i\to 0\ (i=1,....,r)$ in (2.1) and (2.2),in addition to these we replace g by $(-g)$ also set $h_i, g_i \to 0\ (i=1,...,r)$ we get the following more elegant formulas :

$$D^n_{k,\alpha,x}\{x^t\left(x^{\upsilon_1}+a\right)^{\lambda}\left(b-x^{\upsilon_2}\right)^{-\delta}H[z_1x^{\rho_1},...,\mathrm{z}_r\,x^{\rho_r}]\}$$

$$=a^{\lambda}b^{-\delta}x^{t+nk}\sum_{l,m=0}^{\infty}\binom{\lambda}{l}\binom{\delta}{m}\frac{\left(x^{\upsilon_1}/a\right)^l\left(x^{\upsilon_2}/b\right)^m}{l!m!}$$

$$H^{\,0,\mathrm{n}'+n:\mathrm{m}'_{1,}n'_1;....;m'_r,n'_r}_{\,p'+n,q'+n:p'_1,q'_1;....;p'_r,q'_r}\left[\begin{array}{c}z_1x^{\rho_1}\\ \cdot\\ \cdot\\ z_rx^{\rho_r}\end{array}\left|\begin{array}{l}{,}\left(-t-kg-\upsilon_1l-\upsilon_2m;\rho_1,...,\rho_r\right)_{g=0,\mathrm{n}-1},\left(a_j,\alpha_j^{(1)}...\alpha_j^{(r)}\right)_{1,p'}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p'_1};...;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p'_r}\\ \\ \\ \left(\alpha-t-kg-\upsilon_1l-\upsilon_2m;\rho_1,...,\rho_r\right)_{g=0,n-1},\left(b_j,\beta_j^{(1)}...\beta_j^{(r)}\right)_{1,q'}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q'_1};...;\left(d_j^{r},\delta_j^{(r)}\right)_{1,q'_r}\end{array}\right.\right]$$

... (3.1)

$$D^{n}_{k,\alpha,x}\, D^{\mu}_{y}\, \{ x^{t} y^{\lambda} \left(x^{\upsilon_1}+a\right)^{\lambda} \left(b-x^{\upsilon_2}\right)^{-\delta} \left(y^{\upsilon_3}+c\right)^{h} \left(d-y^{\upsilon_4}\right)^{-g}$$
$$H\,[z_1 x^{k_1} y^{\lambda_1},\ldots\ldots, z_r x^{\rho_r} y^{\lambda_r}]\}$$

$$= a^{\lambda} b^{-\delta} c^{h} d^{-g} x^{t+nk} y^{\lambda-\mu} \sum_{l,m,r,s=0}^{\infty} \binom{\sigma}{l}\binom{\delta}{m}\binom{h}{r}\binom{g}{s} \frac{\left(x^{\upsilon_1}/a\right)^{l}\left(x^{\upsilon_2}/b\right)^{m}\left(y^{\upsilon_3}/c\right)^{r}\left(y^{\upsilon_4}/d\right)^{s}}{l!m!r!s!}$$

$$H^{0,\, n'+n+1:\, m'_1, n'_1, \ldots, m'_r, n'_r}_{p'+n+1,\, q'+n+1:\, p'_1, q'_1, \ldots, p'_r, q'_r}$$
$$\left[\begin{matrix} z_1 x^{k_1} y^{\lambda_1} \\ \vdots \\ z_r x^{k_r} y^{\lambda_r} \end{matrix}\middle| \begin{matrix} \left(-\lambda-r\upsilon_3-s\upsilon_4, \lambda_1, \ldots \lambda_r\right), \left(-t-kg-\upsilon_1 l-\upsilon_2 m; k_1, \ldots k_r\right)_{g=0,n-1}, \\ \\ \left(-\lambda+\mu-r\upsilon_3-s\upsilon_4, \lambda_1, \ldots \lambda_r\right), \left(\alpha-t-kg-\upsilon_1 l-\upsilon_2 m, k_1 \ldots k_r\right)_{g=0,n-1}, \end{matrix}\right.$$
$$\left.\begin{matrix} \left(a_j, \alpha_j^{(1)} \ldots \alpha_j^{(r)}\right)_{1,p'} : \left(c_j^{(1)}, \gamma_j^{(1)}\right)_{1,p'_1} \ldots \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p'_r} \\ \\ \left(b_j, \beta_j^{(1)} \ldots \beta_j^{(r)}\right)_{1,q'} : \left(d_j^{(1)}, \delta_j^{(1)}\right)_{1,q'_1} \ldots \left(d_j^{r}, \delta_j^{(r)}\right)_{1,q'_r} \end{matrix}\right]$$

...(3.2)

ACKNOWLEDGEMENT

The author is highly grateful to Dr. Atul Garg forhis valuablehelp and suggestions to improve this paper

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